Statistical error analysis of reactivity measurement

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1. Introduction

The point kinetic model had been used to measure control rod worth without 3D spatial information of neutron flux or power distribution, which causes inaccurate result[1]. Dynamic Control rod Reactivity Measurement (DCRM)[2] was employed to take into account of 3D spatial information of flux in the point kinetics model.

The measured bank worth probably contains some uncertainty such as methodology uncertainty and measurement uncertainty. Those uncertainties may varies with size of core and magnitude of reactivity. The goal of this research is to investigate the effect of core size and magnitude of control rod worth on the error of reactivity measurement using statistics.

2. Methodology

2.1 Data source

For the investigation, control rod worth data were obtained from nuclear startup test for both Westinghouse (WH) type and OPR-1000. The obtained data are arranged by core size and bank worth size in three groups respectively as shown in table 1 and table 2.

Table 1. The data arranged by core size.

Grouped by Core size			
Group	Small	Medium	Large
Relative core size	1.00	1.54	1.61
Power Plant Unit	Kori1,2	Kori3, 4 Yonggwang1,2 Ulchin1,2	Yonggwang3,4,5,6 Ulchin3,4,5 ShinKori1
No. Data	24	66	96

Table 2. The data arranged by bank worth size.

Grouped by Bank Worth			
Group	Small	Medium	Large
Rod Worth(pcm)	200-600	601-900	>900
No. Data	77	82	27

Measured bank worth data were obtained using DCRM while predicted bank worth data were calculated by PHOENIX/ANC for WH type and calculated by DIT/ROCS for OPR-1000.

2.2 DCRM

DCRM has been successfully applied to control rod worth measurement for WH type reactor and OPR-1000. All measurement data used in this paper were obtained by DCRM. Rod worth in DCRM is given by

$$\rho(t_n) = \sum_k \beta_k \left(e^{(\lambda_k+\omega_n)\Delta t_n} B_{n-1,k} + A_{n,k}\right) + \Lambda\omega_n - \bar{S}_0 \frac{\Lambda}{\bar{n}_n} \quad (1)$$

Where; $B_{n,k} = e^{-(\lambda_k+\omega_n)\Delta t_n} B_{n-1,k} + A_{n,k}$,

$A_{n,k} = \frac{\omega_n}{(\lambda_k+\omega_n)}(1 - e^{-(\lambda_k+\omega_n)\Delta t_n})$, and

All kinetics parameters in equation *(1)* are conventional definition used in the reference [1].

In DCRM, Density-to-Response Conversion Factor (DRCF) and Dynamic-to Static Conversion Factor (DSCF) are introduced to account for 3D spatial effect of flux in point kinetics equation.

2.3 Statistical Analysis

Grouped data were tested normality to ensure normal distribution. After normality test, hypothesis test on the mean and variance were performed to check equal mean and equal variance. From this hypothesis test, we can confirm that each group was sampled from the same population. Fig.1 presents the flow process of statistical analysis.

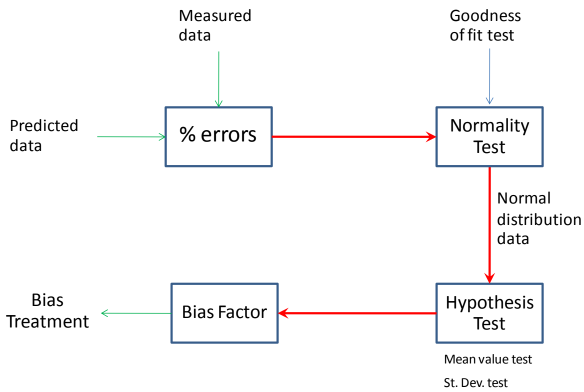

Fig1. Flow process analysis

2.3.1 Normality Test

Normality Test[3] was applied for each group using Kolmogorov-Smirnov test method. The test was performed by Minitab software and the test results are shown in table 3 and table 4 as below;

Table 3. Normality Test for core size data set

Group	*Small*	*Medium*	*Large*
Mean	-3.048	-1.095	-1.305
Standard Deviation	5.488	5.379	5.540
P-Value	0.038	>0.150	>0.150

Table 4. Normality Test for Rod worth data set

Group	*Small*	*Medium*	*Large*
Mean	-1.451	-1.681	-1.273
Standard Deviation	5.998	5.665	5.153
P-Value	>0.150	>0.150	>0.150

At 5% significant level, the P-Value shown in Table 3 and Table 4 indicate normal distribution except the group of small core size. The number of the group of small core size may not be bigger for 5% significant level. However it can be accepted normal distribution when the number of data increases.

2.3.2 Hypothesis Test

Since all group pass normality test, statistics from each group can be considered meaningful in statistical sense. Hypothesis test on mean and variance are performed to find that each group comes from the same population.

(1) One Way ANOVA test[4] at 5 % significant level was performed to check equal-mean hypothesis test using Minitab software for both Core size group and Rod worth group. The result s are shown below;

One-way ANOVA: % error versus Core size Ratio

```
Source            DF      SS     MS     F      P
Core size Ratio    2    70.4   35.2  1.17  0.311
Error            183  5489.4   30.0
Total            185  5559.9
```

One-way ANOVA: % Error versus Worth Group

```
Source        DF      SS    MS     F      P
Worth Group    2     6.6   3.3  0.11  0.897
Error        183  5553.3  30.3
Total        185  5559.9
```

F-value at $f_{0.05}$, (2,183) is 2.9957. Calculated F-values are 1.17 and 0.11 respectively which are smaller than F-value at $f_{0.05}$, (2,183). Therefore, the null hypothesis is acceptable.

(2) The F-test[4] at 5% significant level was performed to test equal-variance hypothesis for each group. The results are shown in table 5 and table 6 for core size group and rod worth group respectively;

Table 5. Variance Test for core size

Group compared	*DF*	*F-value*	*F-calculated*
Small VS. Medium	23,65	1.7025	1.0409
Medium VS. Large	65,95	1.4627	0.9427
Small VS. Large	23,95	1.6575	0.9813

Table 6. Variance Test for Rod worth

Group compared	*DF*	*F-value*	*F-calculated*
Small VS. Medium	76,81	1.4375	1.1210
Medium VS. Large	81,26	1.7792	1.2086
Small VS. Large	76,26	1.7840	1.3549

For all test, F-value at $f_{\alpha}, (\upsilon 1, \upsilon 2)$ is larger than F-calculated so that the null hypothesis for equal-variance is acceptable.

2.3.3 Bias Factor

Bias factor[5] was obtained using follow equation

$$Error^2 = [(a)\ Predicted]^2 - Measured^2 = 0$$

Where; '*a*' is considered as bias value.

The bias factor was applied to predicted data in order to reduce the difference between predicted data and measured data. The results are shown in table 7 and table 8 respectively;

Table 7. Bias factor for Core Size

Core Size Group	*Mean % Error*	*Mean Bias*	*Mean % Error (Bias Treatment)*
Small	-3.05	0.97	-0.26
Medium	-1.09	0.99	-0.28
Large	-1.35	0.99	-0.3

Table 8. Bias factor for Rod worth

Rod Worth Size Group	*Mean % Error*	*Mean Bias*	*Mean % Error (Bias Treatment)*
Small	-1.27	0.99	-0.25
Medium	-1.68	0.99	-0.31
Large	-1.45	0.99	-0.33

3. Conclusions

After statistical analysis, it was confirmed that each group were sampled from same population. It is observed in Table 7 that the mean error decreases as core size increases.

Application of bias factor obtained from this research reduces mean error further.

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